WHAT IS STATISTICAL LEARNING?

Chapter 02 – Part I

Outline

- What Is Statistical Learning?
  - Why estimate f?
  - How do we estimate f?
  - The trade-off between prediction accuracy and model interpretability
  - Supervised vs. unsupervised learning
  - Regression vs. classification problems

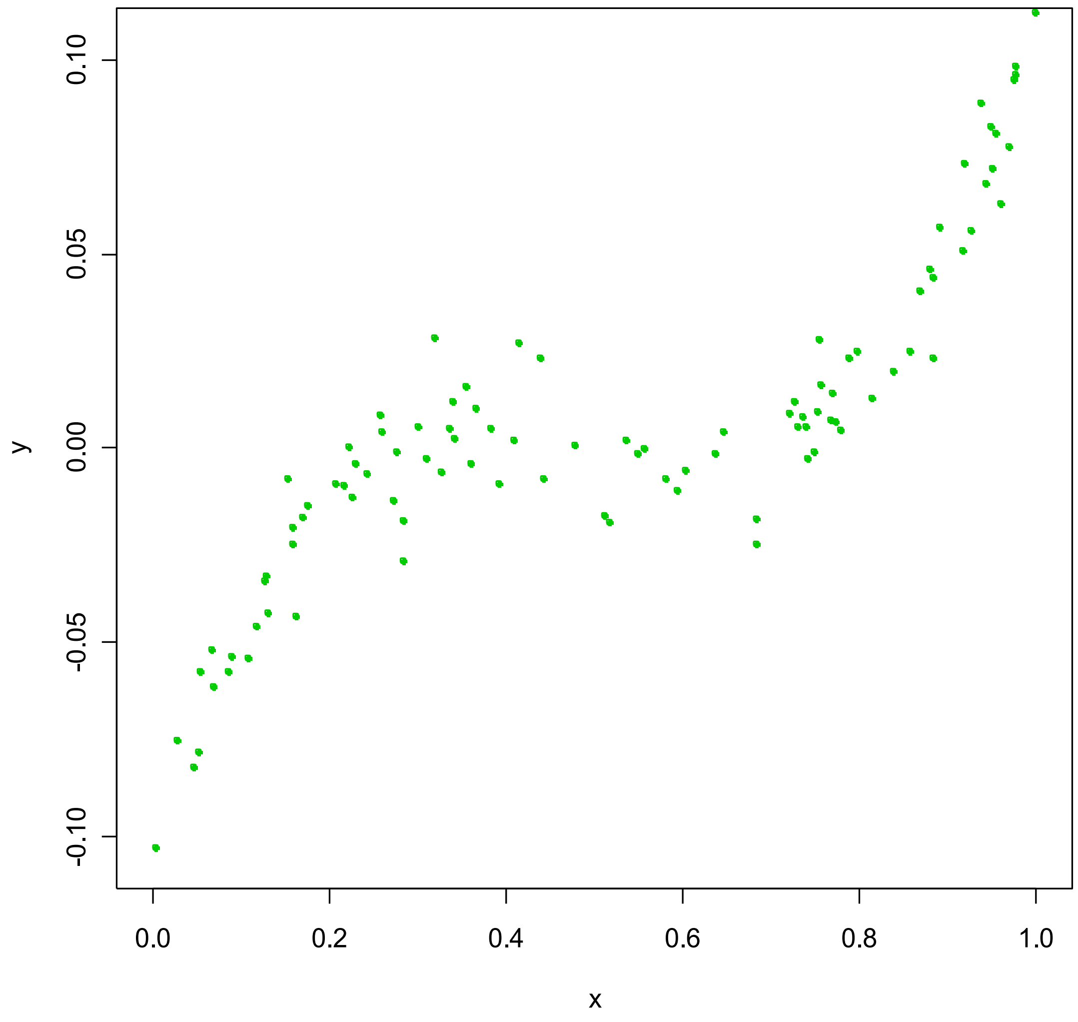
What is Statistical Learning?

- Suppose we observe Y_i and $X_i = (X_{i1}, \ldots, X_{ip})$ for $i = 1, \ldots, n$
- We believe that there is a relationship between Y and at least one of the X's.
- We can model the relationship as

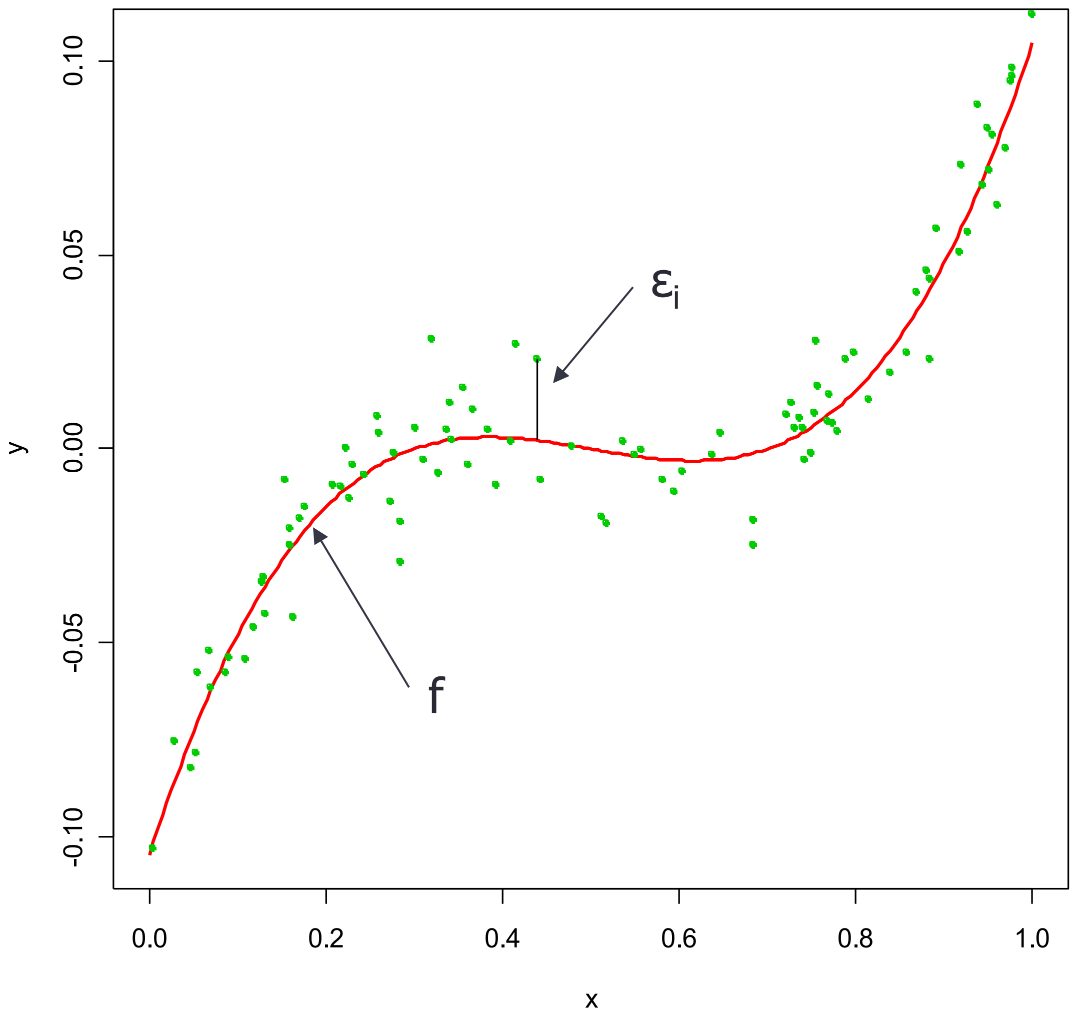
$$Y_i = f(\mathbf{X}_i) + \varepsilon_i$$

- Where f is an unknown function and ε is a random error with mean zero.

A Simple Example

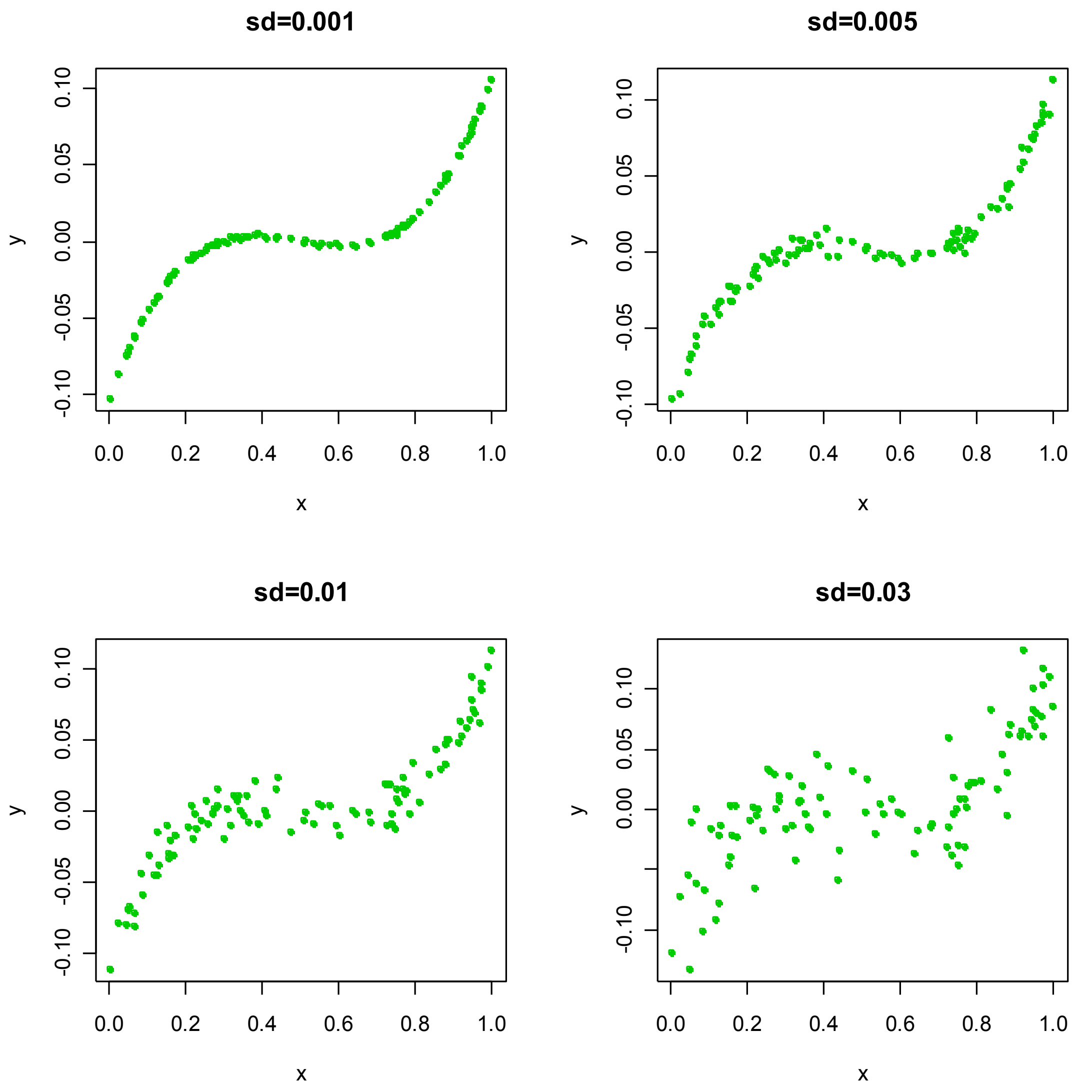


A Simple Example

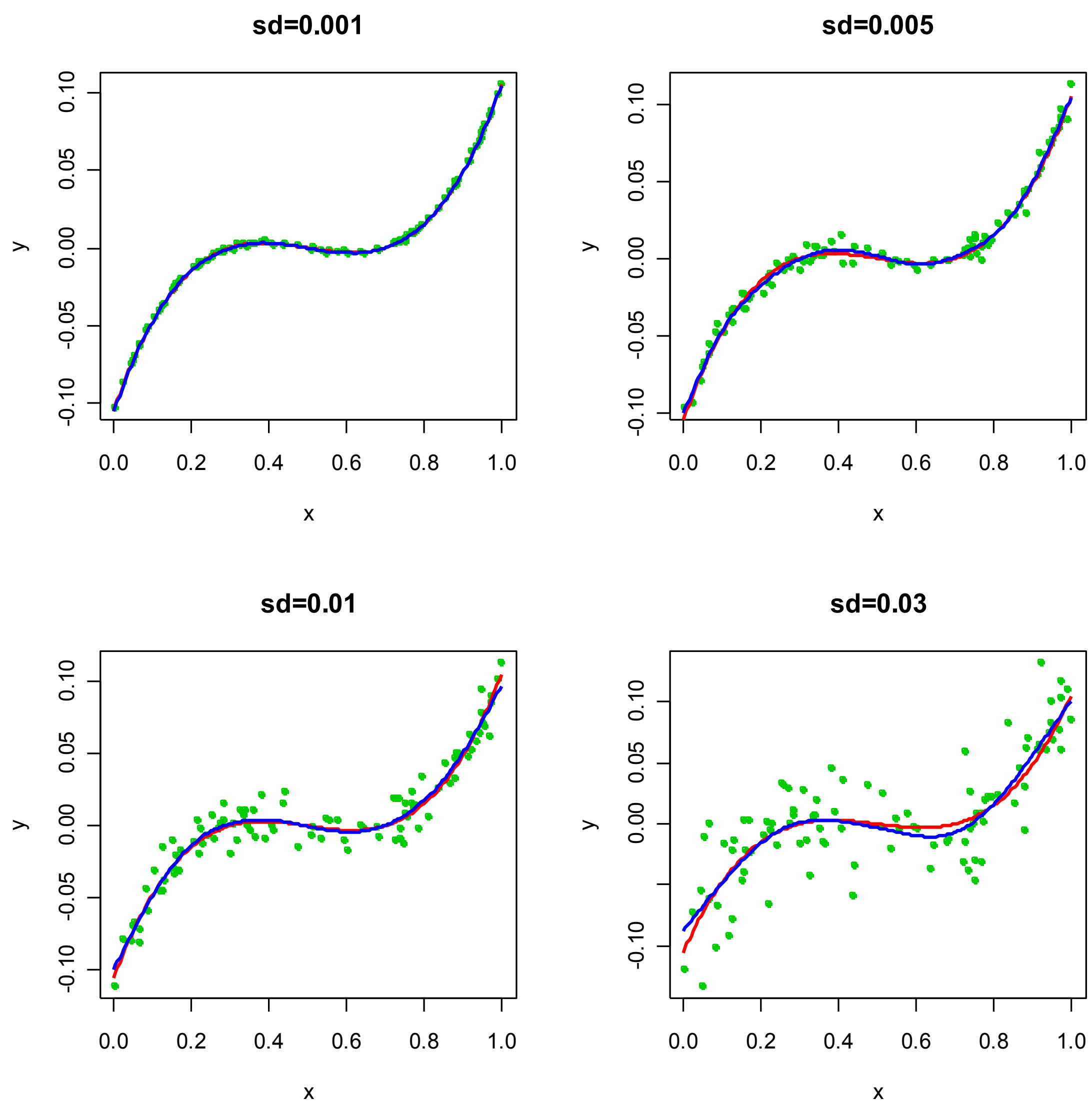


Different Standard Deviations

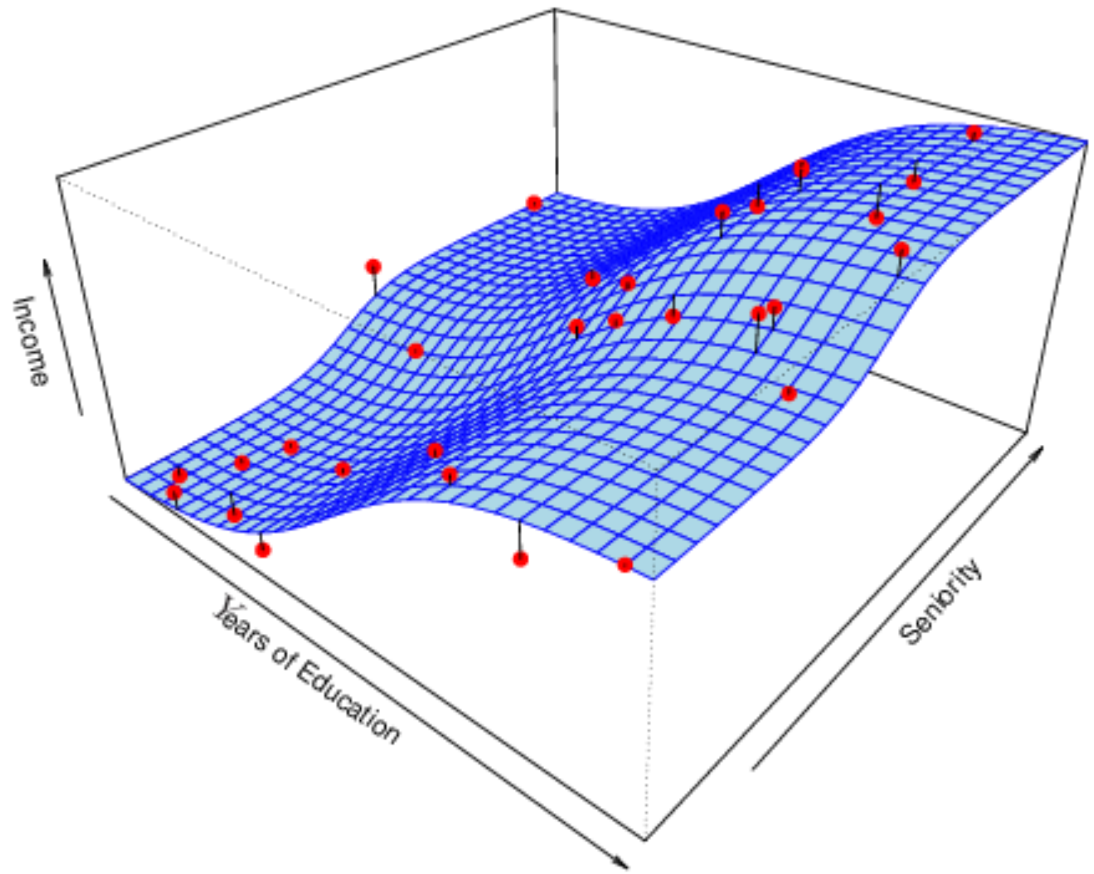
- The difficulty of estimating f will depend on the standard deviation of the ε's.



Different Estimates For f



Income vs. Education Seniority



Why Do We Estimate f?

- Statistical Learning, and this course, are all about how to estimate f.
- The term statistical learning refers to using the data to “learn” f.
- Why do we care about estimating f?
- There are 2 reasons for estimating f,
  - **Prediction** and
  - **Inference.**

1. Prediction

- If we can produce a good estimate for f (and the variance of ε is not too large) we can make accurate predictions for the response, Y, based on a new value of **X**.

Example: Direct Mailing Prediction

- Interested in predicting how much money an individual will donate based on observations from 90,000 people on which we have recorded over 400 different characteristics.
- Don't care too much about each individual characteristic.
- Just want to know: For a given individual should I send out a mailing?

2. Inference

- Alternatively, we may also be interested in the type of relationship between Y and the X's.
- For example,
  - Which particular predictors actually affect the response?
  - Is the relationship positive or negative?
  - Is the relationship a simple linear one or is it more complicated etc.?

Example: Housing Inference

- Wish to predict median house price based on 14 variables.
- Probably want to understand which factors have the biggest effect on the response and how big the effect is.
- For example how much impact does a river view have on the house value etc.

How Do We Estimate f?

- We will assume we have observed a set of **training data**

$$\{(\mathbf{X}_1, Y_1), (\mathbf{X}_2, Y_2), \ldots, (\mathbf{X}_n, Y_n)\}$$

- We must then use the training data and a statistical method to estimate f.
- Statistical Learning Methods:
  - Parametric Methods
  - Non-parametric Methods

Parametric Methods

- It reduces the problem of estimating f down to one of estimating a set of parameters.

- They involve a two-step model based approach

STEP 1:

Make some assumption about the functional form of *f*, i.e. come up with a model. The most common example is a linear model i.e.

$$f(\mathbf{X}_i) = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_p X_{ip}$$

However, in this course we will examine far more complicated, and flexible, models for *f*. In a sense the more flexible the model the more realistic it is.

Parametric Methods (cont.)

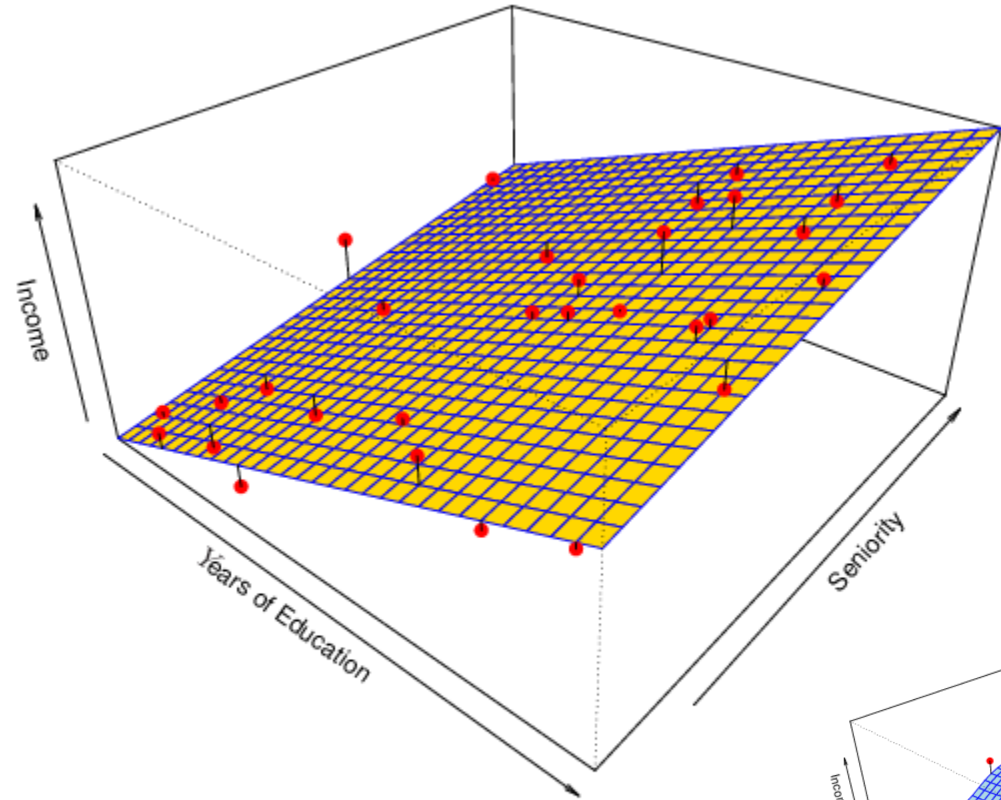
STEP 2:

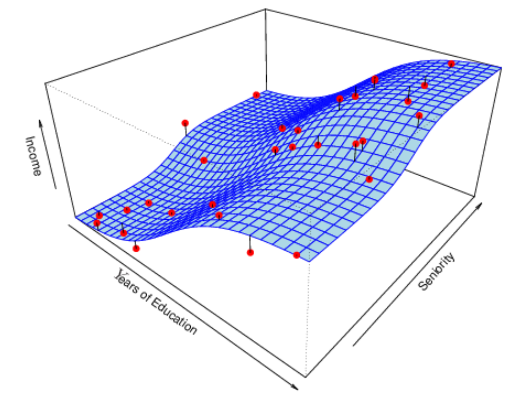
Use the training data to fit the model i.e. estimate *f* or equivalently the unknown parameters such as β_0, β_1, β_2,…, β_p.

- The most common approach for estimating the parameters in a linear model is ordinary least squares (OLS).
- However, this is only one way.
- We will see in the course that there are often superior approaches.

Example: A Linear Regression Estimate

- Even if the standard deviation is low we will still get a bad answer if we use the wrong model.





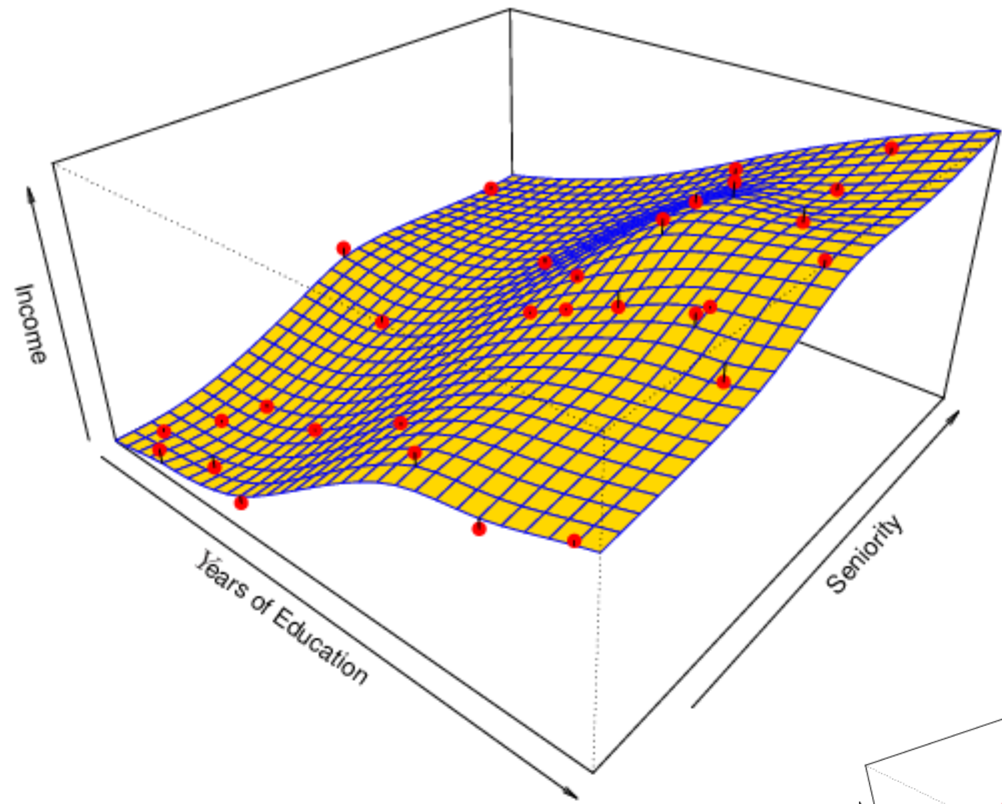
$$f = \beta_0 + \beta_1 \times Education + \beta_2 \times Seniority$$

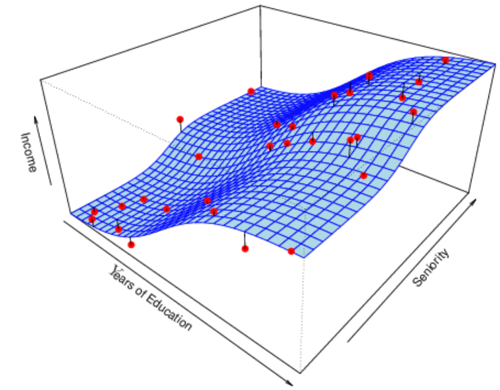
Non-parametric Methods

- They do not make explicit assumptions about the functional form of f.
- Advantages: They accurately fit a wider range of possible shapes of f.
- Disadvantages: A very large number of observations is required to obtain an accurate estimate of f

Example: A Thin-Plate Spline Estimate

- Non-linear regression methods are more flexible and can potentially provide more accurate estimates.





Tradeoff Between Prediction Accuracy and Model Interpretability

- Why not just use a more flexible method if it is more realistic?
- There are two reasons

Reason 1:

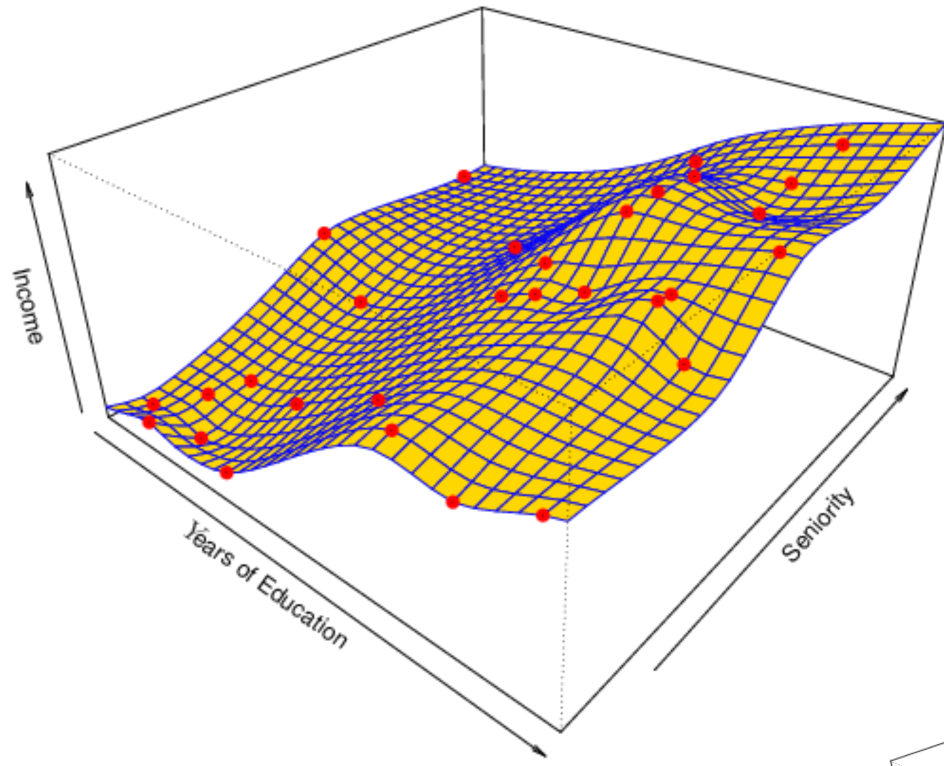
A simple method such as linear regression produces a model which is much easier to interpret (the Inference part is better). For example, in a linear model, β_j is the average increase in Y for a one unit increase in X_j holding all other variables constant.

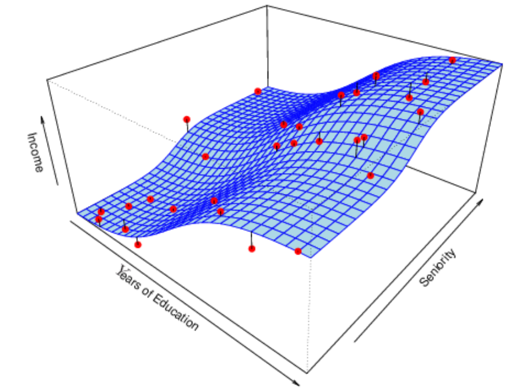
Reason 2:

Even if you are only interested in prediction, so the first reason is not relevant, it is often possible to get more accurate predictions with a simple, instead of a complicated, model. This seems counter intuitive but has to do with the fact that it is harder to fit a more flexible model.

A Poor Estimate

- Non-linear regression methods can also be too flexible and produce poor estimates for *f*.



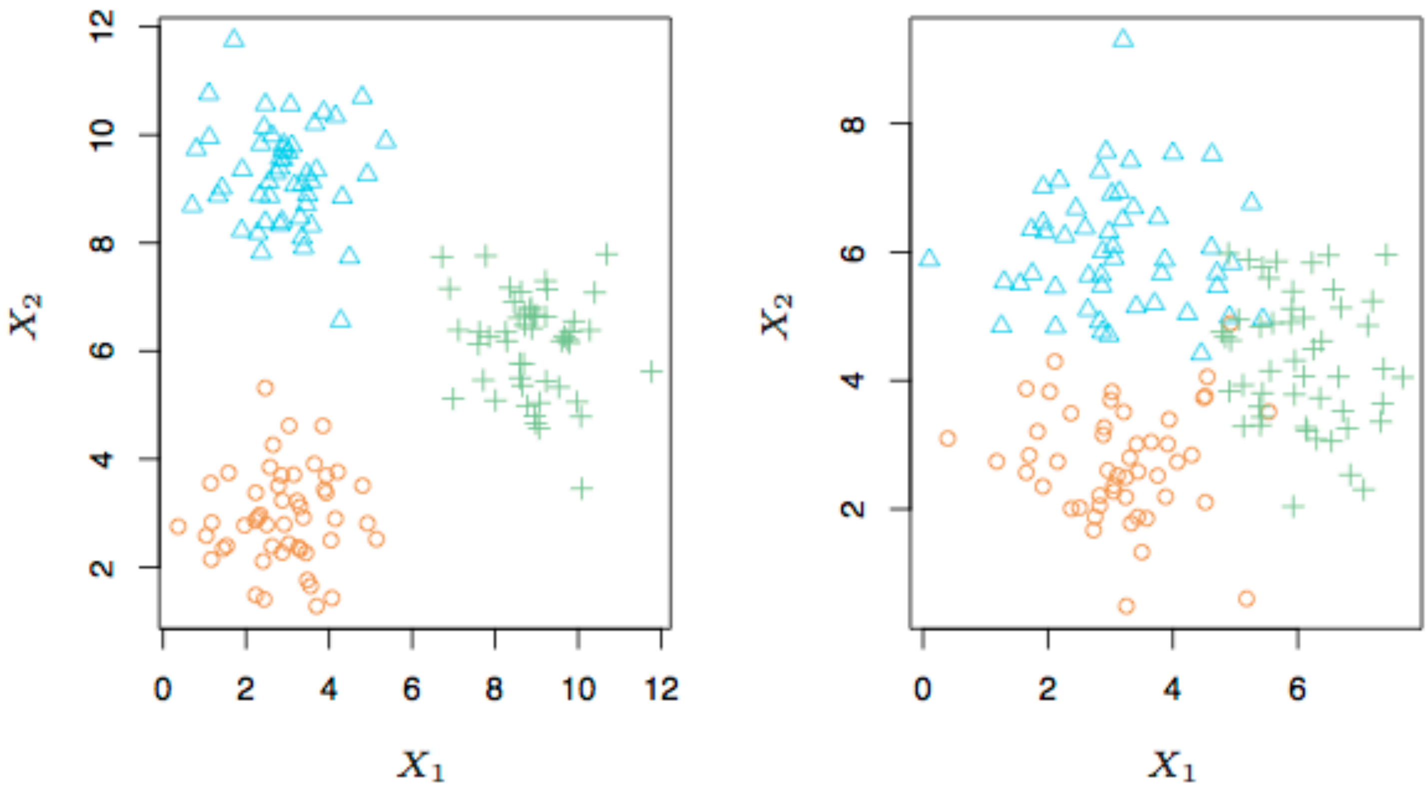


Supervised vs. Unsupervised Learning

- We can divide all learning problems into Supervised and Unsupervised situations
- Supervised Learning:
  - Supervised Learning is where both the predictors, $\mathbf{X}_i$, and the response, Y_i, are observed.
  - This is the situation you deal with in Linear Regression classes (e.g. GSBA 524).
  - Most of this course will also deal with supervised learning.

- <u>Unsupervised Learning:</u>
  - In this situation only the $\mathbf{X}_i$'s are observed.
  - We need to use the $\mathbf{X}_i$'s to guess what Y would have been and build a model from there.
  - A common example is market segmentation where we try to divide potential customers into groups based on their characteristics.
  - A common approach is clustering.
  - We will consider unsupervised learning at the end of this course.

A Simple Clustering Example



Regression vs. Classification

- Supervised learning problems can be further divided into regression and classification problems.
- Regression covers situations where Y is continuous/ numerical. e.g.
  - Predicting the value of the Dow in 6 months.
  - Predicting the value of a given house based on various inputs.
- Classification covers situations where Y is categorical e.g.
  - Will the Dow be up (U) or down (D) in 6 months?
  - Is this email a SPAM or not?

Different Approaches

- We will deal with both types of problems in this course.
- Some methods work well on both types of problem e.g. Neural Networks
- Other methods work best on Regression, e.g. Linear Regression, or on Classification, e.g. k-Nearest Neighbors.